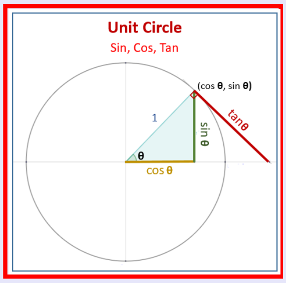
Math 241

Winter 2024

Lecture 3



Class QZ 1

Find the area of a triangle with sides 8cm, 13 cm, and 11 cm.

$\boxed{\text{Box Your final Ans.}}$

Round to a whole #

$S = \frac{8+13+11}{2} = \frac{32}{2} = 16$

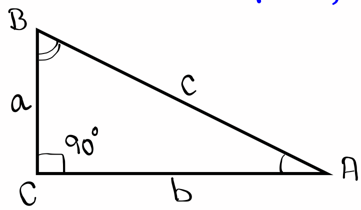
$\text{Area} = \sqrt{16(16-8)(16-13)(16-11)} = \sqrt{16 \cdot 8 \cdot 3 \cdot 5}$

$= \sqrt{1920} \approx 43.818$

$\approx \boxed{44\ cm^2}$

What is Trigonometry?

It is the study of relationships between angles and sides of any right triangle.



$m\angle A + m\angle B = 90°$

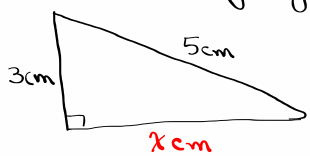
a & b are legs.

c is the hypotenuse

$a^2 + b^2 = c^2$

Pythagorean Thrm.

Find the missing leg.



By Pythagoream thrm

$3^2 + x^2 = 5^2$

$9 + x^2 = 25$

$x^2 = 16$

$x = \pm\sqrt{16}$ $x = \pm 4$

$\boxed{x = 4}$

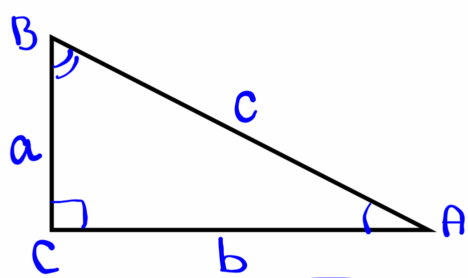
Missing leg is 4cm.

Trig. Functions

Sin → Sine

Cos → Cosine

tan → tangent

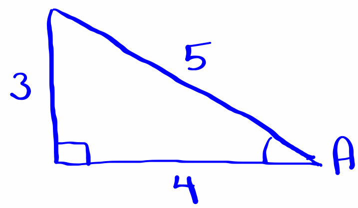


$\boxed{\text{Sin } A = \frac{\text{opposite}}{\text{hypotenuse}}}$ $\boxed{\text{Sin } A = \frac{a}{c}}$

$\boxed{\text{Cos } A = \frac{\text{adjacent}}{\text{Hypotenuse}}}$ $\boxed{\text{Cos } A = \frac{b}{c}}$

$\boxed{\tan A = \frac{\text{opposite}}{\text{Adjacent}}}$ $\boxed{\tan A = \frac{a}{b}}$

Use the right triangle below to find $\sin A$, $\cos A$, and $\tan A$.



Verify $a^2 + b^2 = c^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

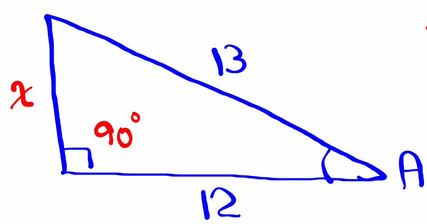
$25 = 25$ ✓

$\sin A = \frac{\text{opp.}}{\text{hyp.}}$ $\quad \sin A = \frac{3}{5}$

$\cos A = \frac{\text{adj.}}{\text{hyp.}}$ $\quad \cos A = \frac{4}{5}$

$\tan A = \frac{\text{opp.}}{\text{adj.}}$ $\quad \tan A = \frac{3}{4}$

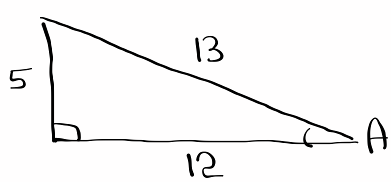
Find $\sin A$, $\cos A$, and $\tan A$ using the shape below.



$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

$x^2 = 25 \rightarrow \boxed{x = 5}$



$\sin A = \frac{\text{opp.}}{\text{hyp.}}$ $\quad \boxed{\sin A = \frac{5}{13}}$

$\cos A = \frac{\text{adj.}}{\text{hyp.}}$ $\quad \boxed{\cos A = \frac{12}{13}}$

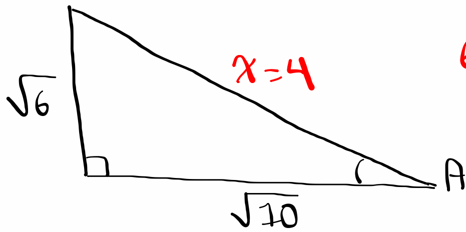
$\tan A = \frac{\text{opp.}}{\text{adj.}}$ $\quad \boxed{\tan A = \frac{5}{12}}$

No mixed numbers.

No decimals.

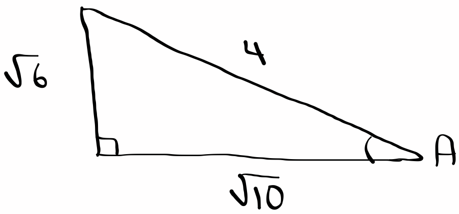
No radicals in the deno.

Find Sin A, Cos A, and tan A using the shape below



$(\sqrt{6})^2 + (\sqrt{10})^2 = x^2$

$6 + 10 = x^2 \qquad x^2 = 16 \qquad \boxed{x=4}$



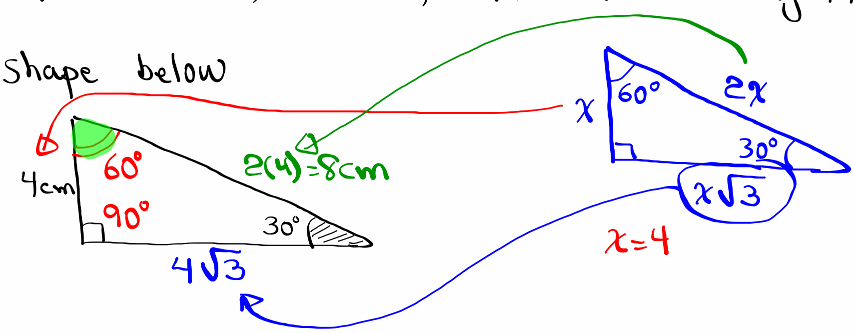
$\sin A = \dfrac{\sqrt{6}}{4}$

$\cos A = \dfrac{\sqrt{10}}{4}$

$\tan A = \dfrac{\sqrt{6}}{\sqrt{10}} = \dfrac{\cancel{\sqrt{2}}\sqrt{3}}{\cancel{\sqrt{2}}\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{\sqrt{15}}{\sqrt{25}} = \dfrac{\sqrt{15}}{5}$

Find Sin 30°, Cos 30°, and tan 60° using the shape below

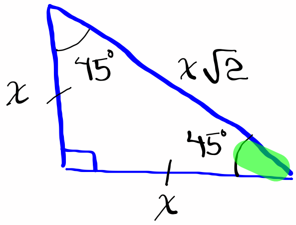


$\sin 30° = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{4}{8} = \dfrac{1}{2}$

$\cos 30° = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{\overset{1}{\cancel{4}}\sqrt{3}}{\underset{2}{\cancel{8}}} = \dfrac{\sqrt{3}}{2}$

$\tan 60° = \dfrac{\text{opp.}}{\text{Adj.}} = \dfrac{\cancel{4}\sqrt{3}}{\cancel{4}} = \boxed{\sqrt{3}}$

Find $\sin 45°$, $\cos 45°$, and $\tan 45°$.



$$\sin 45° = \frac{\cancel{x}}{\cancel{x}\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\cos 45° = \frac{\cancel{x}}{\cancel{x}\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\tan 45° = \frac{x}{x} = \boxed{1}$$

	30°	45°	60°
Sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

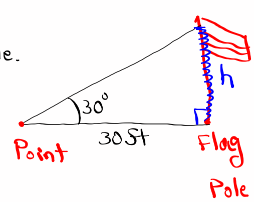
Angle of elevation → when you look up

Angle of depression → when you look down.

my angle of elevation from a point on the ground to the top of flag pole is 30°.

The point is 30 ft from the base of the flag pole.

How tall is the flag pole.



$\tan 30° = \frac{\text{opp.}}{\text{adj.}}$

$\tan 30° = \frac{h}{30}$

Cross-multiply

$h = 30 \cdot \tan 30°$

$= \cancel{30}^{10} \cdot \frac{\sqrt{3}}{\cancel{3}}$

$= 10\sqrt{3}$

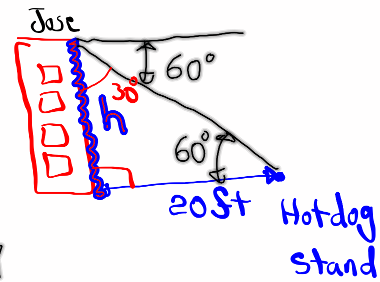
$\approx 17.321 \approx \boxed{17 \text{ ft}}$

Jose is on the top of a building.

His angle of depression is 60° to a hotdog stand on the street.

The hot dog stand is 20 ft from the base of the building.

How tall is the building?

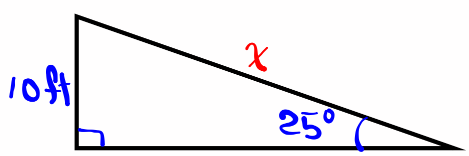


$\tan 60° = \frac{h}{20}$ Cross-Multiply

$h = 20 \cdot \tan 60°$

$= 20 \cdot \sqrt{3} \approx 34.641 \approx \boxed{35 \text{ ft}}$

Find the hypotenuse of the shape below



$\sin 25° = \frac{10}{x}$

Cross-Multiply

$x \cdot \sin 25° = 10$

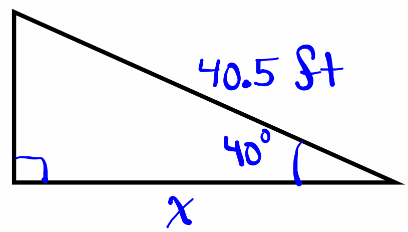
$x = \frac{10}{\sin 25°}$

$x \approx \frac{10}{.423}$

$x \approx 23.641$ $x \approx 24$

Hyp. is about 24 ft.

Find x, round to 1-decimal, using the drawing below



$\cancel{\sin 40^\circ = \frac{opp.}{hyp.}}$

$\cancel{\tan 40^\circ = \frac{opp.}{adj.}}$

Not good options

$\cos 40^\circ = \frac{x}{40.5}$

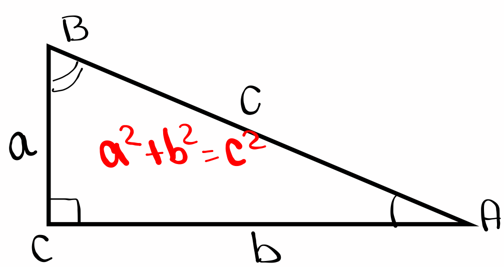
Cross-Multiply

$x = 40.5 \cdot \cos 40^\circ$

$\approx 40.5\,(.766)$

$\approx 31.023 \approx \boxed{31.0 \text{ ft}}$

Prove $\sin^2 A + \cos^2 A = 1$



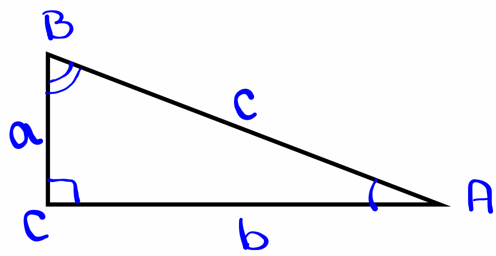
$\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$

$\sin^2 A = \frac{a^2}{c^2}$, $\cos^2 A = \frac{b^2}{c^2}$

$\sin^2 A + \cos^2 A = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = \boxed{1}$

So $\sin^2 A + \cos^2 A = 1$ ✓

Prove $\tan A = \dfrac{\sin A}{\cos A}$



$\tan A = \dfrac{a}{b}$

Divide num. & Deno. by C

$\tan A = \dfrac{\frac{a}{c}}{\frac{b}{c}} = \dfrac{\sin A}{\cos A}$

So $\boxed{\tan A = \dfrac{\sin A}{\cos A}}$

Simplify $(\sin x + \cos x)^2 - 1$

$(\sin x + \cos x)^2 - 1 = (\sin x + \cos x)(\sin x + \cos x) - 1$

$= \boxed{\sin^2 x} + \sin x \cos x + \sin x \cos x \boxed{+ \cos^2 x} - 1$

$= 2 \sin x \cos x \cancel{+1} \cancel{-1}$

$= \boxed{2 \sin x \cos x}$

Simplify

$$(\sin x - \cos x)^2 + 2\sin x\cos x$$

$$= (\sin x - \cos x)(\sin x - \cos x) + 2\sin x\cos x$$

$$= \sin^2 x \underbrace{- \sin x\cos x - \sin x\cos x}\ + \cos^2 x + 2\sin x\cos x$$

$$= 1 - \cancel{2\sin x\cos x} + \cancel{2\sin x\cos x}$$

$$= \boxed{1}$$

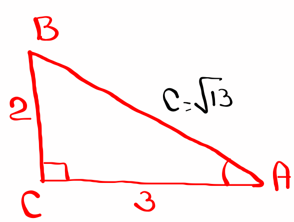
Simplify $\tan x \cdot \cos x$

$$\tan x \cdot \cos x = \frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}$$

$$= \sin x$$

In the right Triangle ABC, $\tan A = \frac{2}{3}$, find $\sin A$ and $\cos A$.

$= \frac{\text{opp.}}{\text{adj.}}$



$$a^2 + b^2 = c^2$$

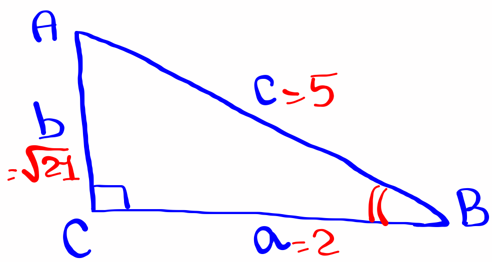
$$4 + 9 = c^2 \qquad c^2 = 13 \qquad \boxed{c = \sqrt{13}}$$

$$\sin A = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{2\sqrt{13}}{13}}$$

$$\cos A = \frac{3}{\sqrt{13}} = \quad = \boxed{\frac{3\sqrt{13}}{13}}$$

In the right triangle ABC, $\cos B = \frac{2}{5}$. $= \frac{\text{adj}}{\text{hyp.}}$

Find $\sin B$ and $\tan B$.



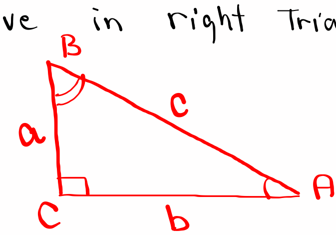
$2^2 + b^2 = 5^2$

$b^2 = 21$ $\quad b = \sqrt{21}$

$\sin B = \frac{\sqrt{21}}{5}$

$\tan B = \frac{\sqrt{21}}{2}$

Prove in right Triangle ABC, $\sin A = \cos B$.



$\sin A = \frac{a}{c}$

$\cos B = \frac{a}{c}$

$\therefore \sin A = \cos B$

A & B are Compl. angles.

$A + B = 90^\circ$

Sine & Co sine are co functions.

$\boxed{\text{Co functions of Complementary angles are equal.}}$

$\sin 20^\circ \approx .342$ $\quad \sin 30^\circ = \frac{1}{2}$ $\quad \sin 90^\circ = 1$

$\cos 70^\circ \approx .342$ $\quad \cos 60^\circ = \frac{1}{2}$ $\quad \cos 0^\circ = 1$

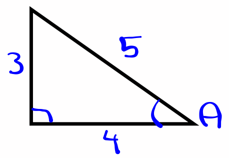
Reciprocal Functions:

Sec → Secant $\quad \operatorname{Sec} A = \dfrac{1}{\operatorname{Cos} A}$

Csc → Co Secant $\quad \operatorname{Csc} A = \dfrac{1}{\operatorname{Sin} A}$

Cot → Cotangent $\quad \operatorname{Cot} A = \dfrac{1}{\tan A}$

Complete the chart below for the given shape



verify

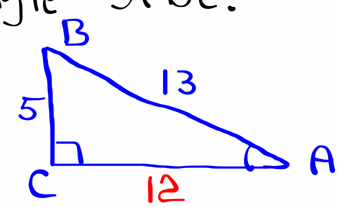
$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

$25 = 25$ ✓

$\operatorname{Sin} A = \frac{3}{5}$	$\operatorname{Csc} A = \frac{5}{3}$
$\operatorname{Cos} A = \frac{4}{5}$	$\operatorname{Sec} A = \frac{5}{4}$
$\tan A = \frac{3}{4}$	$\operatorname{Cot} A = \frac{4}{3}$

Complete the chart below for angle A if $\operatorname{Sin} A = \frac{5}{13}$ in the right triangle ABC.



$\operatorname{Sin} A = \frac{5}{13}$	$\operatorname{Csc} A = \frac{13}{5}$
$\operatorname{Cos} A = \frac{12}{13}$	$\operatorname{Sec} A = \frac{13}{12}$
$\tan A = \frac{5}{12}$	$\operatorname{Cot} A = \frac{12}{5}$

Prove $1 + \tan^2 A = \sec^2 A$

$$1 + \tan^2 A = \boxed{1} + \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} = \frac{\cos^2 A + \sin^2 A}{\cos^2 A}$$

$$= \frac{1}{\cos^2 A} = \sec^2 A \checkmark$$

$$\boxed{1 + \tan^2 A = \sec^2 A}$$

$$\sec^2 A = \frac{1}{\cos^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A}$$

$$= \boxed{\frac{\sin^2 A}{\cos^2 A}} + \boxed{\frac{\cos^2 A}{\cos^2 A}} = \tan^2 A + 1$$

$$= 1 + \tan^2 A$$

Simplify

$$\sin x \cdot \csc x + \cos x \cdot \sec x - \tan x \cdot \cot x$$

$$= \cancel{\sin x} \cdot \frac{1}{\cancel{\sin x}} + \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} - \cancel{\tan x} \cdot \frac{1}{\cancel{\tan x}}$$

$$= 1 + 1 - 1 = \boxed{1}$$

Product of reciprocal functions is equal to 1.

$$\underbrace{\sin 35^\circ \cdot \csc 35^\circ}_{} - \underbrace{\cos 15^\circ \cdot \sec 15^\circ}_{}$$

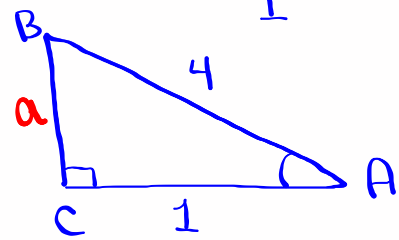
$$= 1 - 1 = \boxed{0}$$

Do not use ϕ for 0.

Complete the chart below for right triangle ACC if $\sec A = 4$.

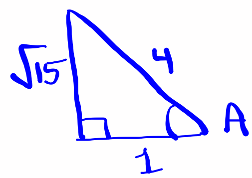
$\sin A = \frac{\sqrt{15}}{4}$	$\csc A = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$
$\cos A = \frac{1}{4}$	$\sec A = 4$
$\tan A = \sqrt{15}$	$\cot A = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

$\sec A = 4 = \frac{4}{1}$

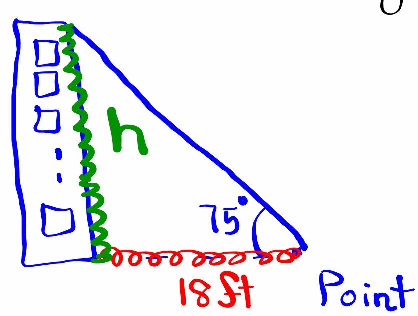


$a^2 + 1^2 = 4^2$

$a = \sqrt{15}$



From a point on the street, the angle of elevation to the top of the building is 75°. Point is 18 ft from the base of the building. How tall is the building?



$\tan 75° = \frac{h}{18}$

Cross-Multiply

$h = 18 \cdot \tan 75° \approx 67.177 \approx \boxed{67 \text{ ft}}$

Now what about negative angles?

$\sin(-\alpha) = -\sin\alpha$ $\qquad$ $\csc(-\alpha) = -\csc\alpha$

Alpha

$\cos(-\alpha) = \cos\alpha$ $\qquad$ $\sec(-\alpha) = \sec\alpha$

$\tan(-\alpha) = -\tan\alpha$ $\qquad$ $\cot(-\alpha) = -\cot\alpha$

Find $\sin(-25^\circ) \approx -.423$

$\sin 25^\circ \approx .423$

Find $\cos(-75^\circ) \approx .259$

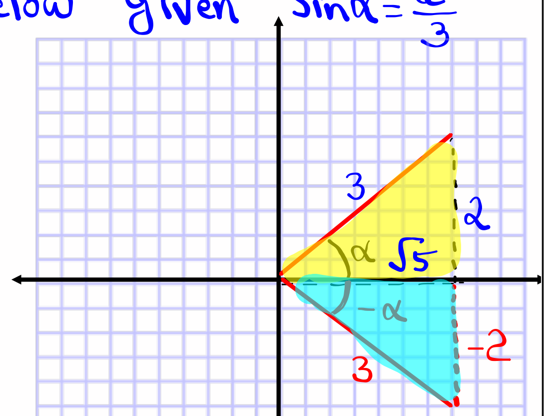
$\cos(75^\circ) \approx .259$

Find $\tan(-15^\circ) \approx -.268$

$\tan 15^\circ \approx .268$

Complete the chart below given $\sin\alpha = \frac{2}{3}$ and α is in QI.

$\sin\alpha = \frac{2}{3}$	$\csc\alpha = \frac{3}{2}$
$\cos\alpha = \frac{\sqrt{5}}{3}$	$\sec\alpha = \frac{3\sqrt{5}}{5}$
$\tan\alpha = \frac{2\sqrt{5}}{5}$	$\cot\alpha = \frac{\sqrt{5}}{2}$



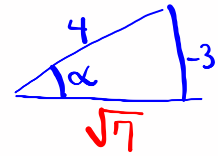
$\sin(-\alpha) = \frac{-2}{3}$

$\cos(-\alpha) = \frac{\sqrt{5}}{3}$

$\tan(-\alpha) = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

Suppose $\sin\alpha = \frac{-3}{4}$ Find

Missing info.



$\sin(-\alpha) = -\sin\alpha = -\left(\frac{-3}{4}\right) = \frac{3}{4}$

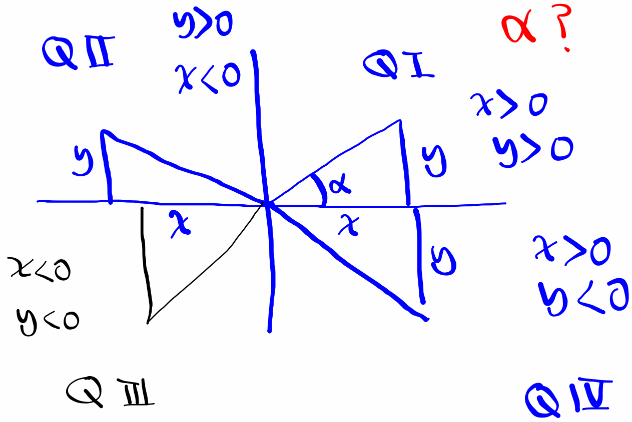
$\cos(-\alpha) = \cos\alpha =$

where is α?

$\tan(-\alpha)$

We visit unit Circle.

S	A
T	C



Distance Formula between two points:

$P_1(x_1, y_1)$, $P_2(x_2, y_2)$

$$d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Find the distance between $P_1(-2, 3)$ and $P_2(4, 6)$.

$$d = \sqrt{(-2-4)^2 + (3-6)^2}$$

$$= \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5} \approx \boxed{6.7}$$

Solve $3x^2 - 8 = 10x.$

$3x^2 - 8 - 10x = 0$

$3x^2 - 10x - 8 = 0$

Method I: Factoring

$3x^2 - 10x - 8 = 0$

-24

Product: -24

Sum: -10

$-12, 2$

$3x^2 - 12x + 2x - 8 = 0$

$3x(x-4) + 2(x-4) = 0$

$(x-4)(3x+2) = 0$

$x - 4 = 0 \qquad 3x + 2 = 0$

$x = 4 \qquad x = \frac{-2}{3}$

$3x^2 - 10x - 8 = 0$

Method II: Q-Formula

$a = 3 \quad b = -10 \quad c = -8$

$b^2 - 4ac = (-10)^2 - 4(3)(-8) = 196$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{196}}{6} = \frac{10 \pm 14}{6}$

$x = \frac{10+14}{6} = \frac{24}{6}$

$x = \frac{10-14}{6} = \frac{-4}{6}$

$\left\{ \frac{-2}{3}, 4 \right\}$ ✓ ✓

Class QZ 2

Solve $9x^2 + 4 = 12x$ by quadratic formula.

$9x^2 + 4 - 12x = 0$

$9x^2 - 12x + 4 = 0$

$a = 9 \quad b = -12 \quad c = 4$

$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-12) \pm \sqrt{0}}{2(9)} = \frac{12 \pm 0}{18} = \frac{12}{18} = \boxed{\frac{2}{3}}$

$\left\{ \frac{2}{3} \right\}$